Online Ad Auctions

Investigating the Mechanics Behind Search Engine Revenue

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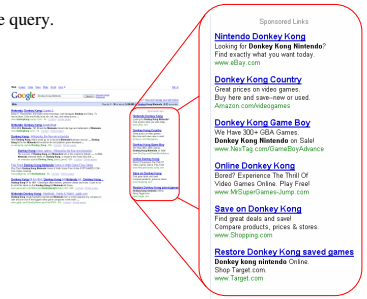
Stanford University
CURIS
Summer 2007

Introduction

A typical search engine's results page has two separate sections:
•The results of the search query
•A list of advertisements relevant to the query.



A sponsored search auction is a process used by search engines to determine:
•The advertisements displayed in response to a search query.
•The price charged to each advertiser.

A search advertising marketplace can be modeled as an auction. Each advertiser is a bidder and the search engine is the auctioneer. The advertising slots are the goods being auctioned. Every time a search is conducted, a quick, real-time auction occurs to decide what ads to display.

Sponsored search auctions are immensely important to search engines. They are currently used by Yahoo!, Google, Microsoft, Overture, and others. In 2005, roughly $7 billion was generated in these auctions. 98% of Google's revenue is said to come from these auctions.

Model

•b_i represents the bid of bidder i
•v_i represents the true value bidder i expects to derive from a click on his ad
•For a specific keyword, there are n advertisers bidding for k slots, with $k < n$. The slots are numbered $1,\ldots,k$ from top to bottom. For simplicity, we number the bidders $1,\ldots,n$ from highest true value to lowest true value.
•The click-through rate (*CTR*) of a slot is the probability that an ad in that slot will be clicked. The click-through rate of slot i can be modeled by the formula $CTR_i = \partial^{i-1}$, where $0<\partial<1$. We denote the CTR of slot i as θ_i.

Generalized Second-Price Auction (GSP)

In a second-price auction for a single item, the bidder who bids the highest is awarded the item and has to pay the bid of the second highest bidder. This auction is truthful: bidders get the highest utility by bidding their true value for the item being auctioned.

A sponsored search auction differs from a traditional second-price auction because multiple goods (advertising slots) are being auctioned simultaneously. The second-price auction can be generalized to a GSP sponsored search auction by ranking the bidders and having each bidder pay the bid of the bidder ranked immediately below him. In this way, it is superficially similar to the single item second-price auction.

	Slot	Bid ($)	Price ($)
Bidder 1	1	1.90	1.55
Bidder 2	2	1.55	1.50
Bidder 3	3	1.50	1.20
Bidder 4	-	1.20	-

Truthfulness

Unlike the second-price auction, a GSP auction is not truthful. Specifically, a bidder might get better utility by shaving his bid, thus moving to a worse slot with a much lower price. Consider the following example:

- Three bidders have values 200, 180, and 100.
- Two slots have *CTR*s 0.5 and 0.4.
- If all bid truthfully, the top bidder's utility in the second slot is 0.5×(200-180)=10.
- If he undercuts the middle bidder and gets the second slot, his utility is 0.4×(200-100)=40.

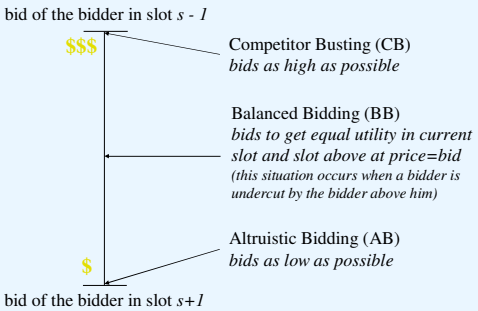
Sponsored search auctions are not inherently untruthful. VCG auctions, a truthful type of auction from classical economic theory, can be used in a sponsored search setting. See the VCG *vs* GSP Revenue section for more information.

Bidding Strategies

As explained above, a bidder is not necessarily motivated to bid his true value. Assume a bidder has determined that he receives the highest possible utility when placed in slot s. To get placed into slot s, he should bid in the range

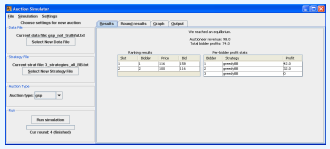
(bid of bidder in slot s+1, bid of bidder in slot s-1).

The value he bids within this range is determined by his bidding strategy.



Auction Simulator

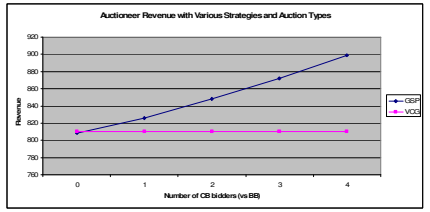
We wrote a tool that can simulate sponsored search auctions. It features:
• Support for multiple auction types (GSP, first-price, VCG)
• Support for a variety of bidding strategies (e.g. CB, BB, AB)
• Cycle/equilibrium detection
• A graphical interface to view per-round information, bidder utilities in different slots, and overall auction statistics



VCG *vs* GSP Revenue

Because a GSP auction is not truthful, bidders often shave their bids in order to be assigned a lower slot. The auction can be easily modified so that bidders pay the VCG payments, in which case it is truthful. The natural question is which type of auction produces the highest revenue for the auctioneer. Although bidders will bid more in a VCG auction, the payments for a given bid are lower. How do these two opposing factors weigh up?

The answer is that, given a reasonable assumption about bidding strategies, the GSP auction gives higher revenue.



If bidders start bidding altruistically, however, the VCG auction might produce higher revenue than the GSP auction.

Equilibrium Properties of BB/CB Auctions

Any auction involving only CB and BB bidders has at most one equilibrium. Equilibrium revenue for these strategy types is always weakly greater than the revenue of the VCG payments. Furthermore, as bidders switch from BB to CB, equilibrium revenue monotonically increases. The exact formula for the equilibrium bids, if an equilibrium exists, is included below.

$$b_i = \begin{cases} v_i & \text{if } i \geq k+1 \\ v_i & \text{if } i \leq k \text{ and player } i \text{ follows the CB strategy} \\ v_i - \frac{\theta_i}{\theta_{i-1}}(v_i - b_{i+1}) & \text{if } i \leq k \text{ and player } i \text{ follows the BB strategy} \end{cases}$$

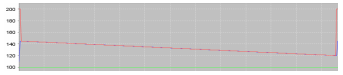
Cycle Properties of BB/CB Auctions

Consider an auction involving only BB/CB bidders at equilibrium. Assume bidder i switches from BB to CB. Then there are two possibilities:

1. Bidder *i-1* still prefers his slot, despite the fact that the price has increased. In this case, the auction immediately reaches a new equilibrium, and revenue increases.
2. Bidder *i-1* prefers to drop down a slot. If bidder *i-1* is using the BB strategy, the resulting cycle obeys the following pattern:

If $3v_{i-1} > 4v_i$, this cycle type results in higher revenue for these two slots than the initial equilibrium.

If bidder *i-1* is using the CB strategy, the resulting cycle obeys the following pattern:



This cycle type results in lower revenue for these two slots than the initial equilibrium.